

Answer all of the following questions.

14 points for each question from 1-6 , question 7 is worth 16 points .

Calculators are not allowed

1. Evaluate the following limits (if they exist)

(a) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{4 \sin^4(\frac{x}{2}) - 1}{x - \frac{\pi}{2}}$

(b) $\lim_{x \rightarrow 0} \frac{x(\sin x)}{(1 - \cos 2x)}$

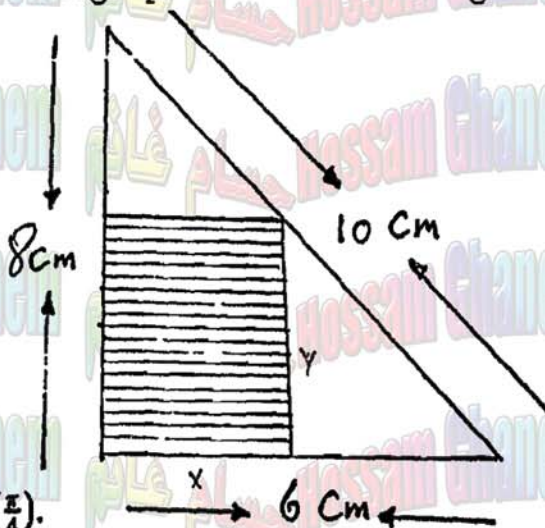
(a) Find the equation for the tangent to the graph of $y + \sqrt{y} \sin x + x \cos x = 1$, at $x = 0$.

(b) Use differential to find an approximate value for $\sqrt{(1.04)^3 + 3} + (1.04)^{\frac{3}{2}}$

3. (a) Test the continuity and differentiability of the function

$$f(x) = \begin{cases} x^2 + 2x + 2 & , \text{if } x < 0 \\ 2 + \sin x & , \text{if } x \geq 0 \end{cases}$$

(b) A rectangle is to be inscribed inside a right triangle of sides 6cm, 8cm and 10cm. Find the dimensions of the rectangle positioned as in the figure so that its area is maximum.



4. (a) Let $f(x) = D_x \left(\int_{\cos x}^{\sin x} t^3 dt \right)$. Find $f(\frac{\pi}{4})$.

(b) Find the average value of the function $f(x) = \sqrt{1+x}$ on the interval $[3, 8]$.

5. (a) Let $\int_1^x f(t) dt = 4x + \frac{2}{3}(x-1)^{\frac{3}{2}}$. Find all numbers c in the interval $(1, 5)$ that satisfies the Mean Value Theorem.

(b) Let $f(x) = \sec^4 \left(\frac{2\sqrt{x+1}}{x^2+5} \right)$, find $f'(x)$.